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Unsteady Free Convection MHD Flow of an Incompressible Electrically Conducting Viscous Fluid through Porous Medium between Two Vertical Plates

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ABSTRACT
In this paper we investigate unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid through porous medium under the influence of uniform transverse magnetic field between two heated vertical plate with one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by the Integral Transform Technique. The obtained results of velocity and temperature distributions are shown graphically and are discussed on the basis of it. The effects of Hartmann number, Darcy parameter, Prandtl number and the decay factor, and effects of adiabatic plate on the velocity and temperature fields are discussed.
Keywords: MHD flow, Unsteady Flow, Adiabatic Plate, Heat Transfer, Darcy parameter

I. Introduction

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid through porous medium has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled electively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field. The unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal plates, lower plate being a stretching sheet and upper being porous was studied by Sharma and Kumar (1998) investigated the unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal plates, lower plate being a stretching sheet and upper being porous. Borkakati and Chakrabarty (2000) unsteady free convection MHD flow between two heated vertical plates. Ray *et al*. (2001) studied the problem of "on some unsteady MHD flows of a second order fluid over a plate". The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account the viscous dissipative heat under the influence of a uniform transverse magnetic field is discussed by Sreekant *et al*. (2001), Gourla and Katoch (1991) studied an unsteady free convection MHD flow between two heated vertical plates. But, they did not discuss about the thermodynamic case on the boundary condition on which the plate is adiabatic. Here our aim is to analyze the unsteady free convection magnetohydrodynamic flow of an incompressible and electrically conducting fluid past between two heated vertical plates in presence of the transverse magnetic field where the temperature of one of the plates changes while the other plate is adiabatic.

In view of these, we studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid through porous medium under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

II. The reduced differential transform method (RDTM)

Let us consider free convective unsteady MHD flow of a viscous incompressible electrically conducting fluid through porous medium between two heated vertical parallel plates. Let x-axis be taken along the vertically upward direction through the central line of the channel and the y-axis is perpendicular to the x-axis. The plates of the channel are kept at $y = \pm h$ distance apart. A uniform magnetic field B_0 is applied in the plane of y-axis and perpendicular to the both x axis and y-axis. $u^{'}$ is in the direction of velocity of fluid, along the x-axis and $v^{'}$ is the velocity along the y-axis. Consequently $u^{'}$ is a function of $y^{'}$ and $t^{'}$, but $v^{'}$ is independent of $y^{'}$. The fluid is assumed to be of low conductivity, such that the induced magnetic field is negligible.

In order to derive the equations of the problem, we assume that the fluid is finitely conducting and the viscous dissipation the Joule heats are neglected. The polarization effect is also neglected.

At time $\tau > 0$, the temperature of the plate at $y = h$ changes according to the temperature function: $T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where T_w and T_0 are the temperature at the plates $y = \eta$ and at $y = -h$ respectively, and $n^{'}(\geq 0)$ is a real number, denoting the decay factor.

Hence the flow field is seen to be governed by the following equations

Equation of Continuity: $$\frac{\partial v^{'}}{\partial y^{'}} = 0 \tag{2.1}$$

Equation of motion: $$\frac{\partial u'}{\partial t^{'}} = \nu \frac{\partial^2 u^{'}}{\partial y^{'2}} + g\beta(T^{'} - T_0) - \frac{\sigma B_0^2}{\rho} u^{'} - \frac{\nu}{k} u^{'} \tag{2.2}$$

Equation of energy: $$\frac{\partial T'}{\partial t^{'}} = \frac{K}{\rho C_p} \frac{\partial^2 T^{'}}{\partial y^{'2}} \tag{2.3}$$

Where

$\rho =$ density of the fluid,
$B_0 =$ uniform magnetic field applied transversely to the plate,
$\sigma =$ electrical conductivity of the fluid,
$\nu =$ co-efficient of kinematics viscosity,
$K =$ thermal conductivity of the fluid,
$C_p =$ specific heat at constant pressure,
$\beta =$ co-efficient of theraml expansion,
$g =$ acceleration due to gravity,
$T^{'} =$ temperature of the fluid,
$[-h, h] =$ space between the plates,
$n^{'}$ or $n =$ decay factor,
$T_0 =$ initial temperature of the plates and liquid,
$T_w =$ wall temperature,
$\mu =$ dynamic viscosity of the fluid,
$D^{-1} =$ Darcy parameter

The initial and boundary conditions for the problem are:

$$t^{'} = 0 : u' = 0, T^{'} = T_0 \qquad \forall\, y^{'} \in [-h, h]$$

$$t^{'} > 0 : u' = 0, T^{'} = T_0 + (T_w - T_0)(1 - e^{-n^{'}t^{'}}) \quad for\ y^{'} = +h$$
$$: u' = 0, \frac{\partial T'}{\partial y^{'}} = 0 \qquad for\ y^{'} = -h \qquad (2.4)$$

We now introduce the following non-dimensional quantities:

$$u = \frac{\nu u^{'}}{\beta g h^2 (T_w - T_0)}, y = \frac{y^{'}}{h}, T = \frac{T^{'} - T_0}{T_w - T_0},$$
$$t = \frac{\nu t^{'}}{h^2}, P_r = \frac{\mu C_p}{K}, n = \frac{h^2 n^{'}}{\nu}, M = B_0 h \sqrt{\frac{\sigma}{\mu}}, D^{-1} = \frac{h^2}{k} \qquad (2.5)$$

Where Pr is the Prandtl number and M is the Hartmann number.

Using the quantities (2.5) in the equations (2.2) and (2.3),we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (M^2 + D^{-1})u + T \qquad (2.6)$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} \qquad (2.7)$$

Under the above non-dimensional quantities, the corresponding boundary conditions redues to

$$t = 0 : u = 0, T = 0 \qquad \forall\, y \in [-1, 1]$$

$$t > 0 : u = 0, T = 1 - e^{-nt}, \quad for\ y = +1$$

$$: u = 0, \frac{\partial T}{\partial y} = 0\ , \quad for\ y = -1 \qquad (2.8)$$

III. Solution of the equations

Now, taking the Laplace Transform of equations (2.6) and (2.7), we obtain

$$\frac{d^2 \bar{u}}{dy^2} - (M^2 + D^{-1} + s)\bar{u} = -\bar{T} \qquad (3.1)$$

$$\frac{d^2 \bar{T}}{dy^2} - \Pr s \bar{T} = 0 \qquad (3.2)$$

Similarly,using Laplace Transform on the boundary conditions (2.8),we get

$$\bar{u}(\pm 1, s) = 0, \quad \bar{T}(1, s) = \frac{n}{s(s+n)}, \quad \frac{d\bar{T}(-1, s)}{dy} = 0 \qquad (3.3)$$

Since the equations (3.1) and (3.2) are 2nd order differential equations in $\bar{u}$ and $\bar{T}$, the solutions of the equations by use condition (3.3) are obtain as

$$\bar{u} = \frac{n\,Sinh\sqrt{M^2 + D^{-1} + s(y-1)}}{s(s+n)(M^2 + D^{-1} + s - s\,\mathrm{Pr})Sinh\,2\sqrt{M^2 + D^{-1} + s}}$$
$$+\frac{n\,Cosh\sqrt{s\,\mathrm{Pr}(y+1)}}{s(s+n)(M^2 + D^{-1} + s - s\,\mathrm{Pr})Cosh2\sqrt{s\,\mathrm{Pr}}}$$
$$-\frac{n\,Sinh\sqrt{M^2 + D^{-1} + s(y+1)}}{s(s+n)(M^2 + D^{-1} + s - s\,\mathrm{Pr})Sinh\,2\sqrt{M^2 + D^{-1} + s}}$$
$$+\frac{n\,Cosh\sqrt{s\,\mathrm{Pr}(y+1)}}{s(s+n)(M^2 + D^{-1} + s - s\,\mathrm{Pr})Cosh2\sqrt{s\,\mathrm{Pr}}} \tag{3.4}$$

$$\bar{T} = \frac{n}{s(s+n)}\frac{Cosh\sqrt{s\,\mathrm{Pr}\,(y+1)}}{Cosh\,2\sqrt{s\,\mathrm{Pr}}} \tag{3.5}$$

Again,using the inverse Laplace Transform on both sides of the equations (3.2) and (3.3),we obtain

$$u = \frac{1}{M^2 + D^{-1}}[1 + \frac{Sinh\sqrt{M^2 + D^{-1}}\,y}{Sinh\sqrt{M^2 + D^{-1}}}] + \frac{e^{-nt}}{n(1-\mathrm{Pr}) - M^2 + D^{-1}}(\frac{Sin\sqrt{n - M^2 + D^{-1}(y-1)}}{Sin\,2\sqrt{n - (M^2 + D^{-1})}\,Cos\,2\sqrt{n\,\mathrm{Pr}}}$$
$$+\frac{Cos\sqrt{n\,\mathrm{Pr}\,(y+1)}}{Cos\,2\sqrt{n\,\mathrm{Pr}}}) + \frac{2\,Sin(\sqrt{M^2 + D^{-1}})\sqrt{\frac{\mathrm{Pr}}{1-\mathrm{Pr}}(y+1)}}{Sin\,2(\sqrt{M^2 + D^{-1}})\sqrt{\frac{\mathrm{Pr}}{1-\mathrm{Pr}}}}e^{(\frac{M^2 + D^{-1}}{1-\mathrm{Pr}})t} \tag{3.6}$$

$$T = 1 - \frac{Cosh\sqrt{n\,\mathrm{Pr}\,(y+1)}\,e^{-nt}}{Cosh\,2\sqrt{n\,\mathrm{Pr}}} \tag{3.7}$$

IV. Results and Discussions

Fig.1 shows the variation of velocity u with Hartmann number M for Pr = 0.25, n =1 and t =0.01. It is found that the velocity u decreases with increasing M .
Fig.2 shows the variation of velocity u with decay parameter n for Pr = 0.25 , M = 2 and t =0.01
It is observed that the velocity u decreases with an increase in n .
Fig.3 shows the variation of velocity u with Prandtl number Pr for n =1 , M = 2 and t =0.01. It is noted that the velocity u decreases with increasing Pr .
Fig. 4 shows the variation of velocity u with darcy parameter D^{-1} with decay parameter n for Pr = 0.25 and t =0.01 It is found that the velocity u decreases with increasing D^{-1}.
Fig. 5 shows the variation of temperature T with Prandtl number Pr for n =1 and t =0.01. It is observed that the temperature T decreases with increasing Prandtl number Pr .
Fig. 6 shows the variation of temperature T with decay parameter n for Pr=0.25 and t =0.01. It is observed that the temperature T decreases with increasing decay parameter n.

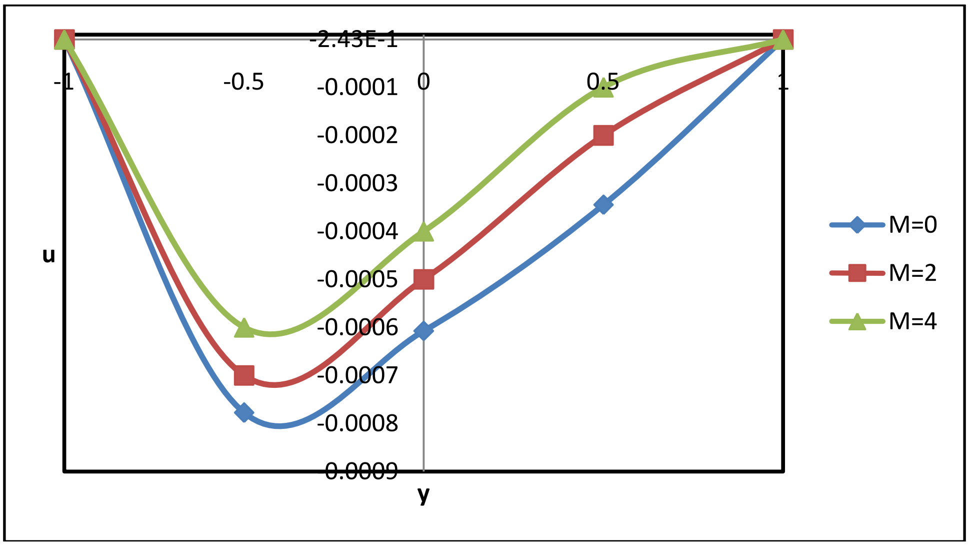

Fig. 1 The variation of velocity u with Hartmann number M φορ Πρ = 0.25 , ν=1 ανδ τ=0.01.

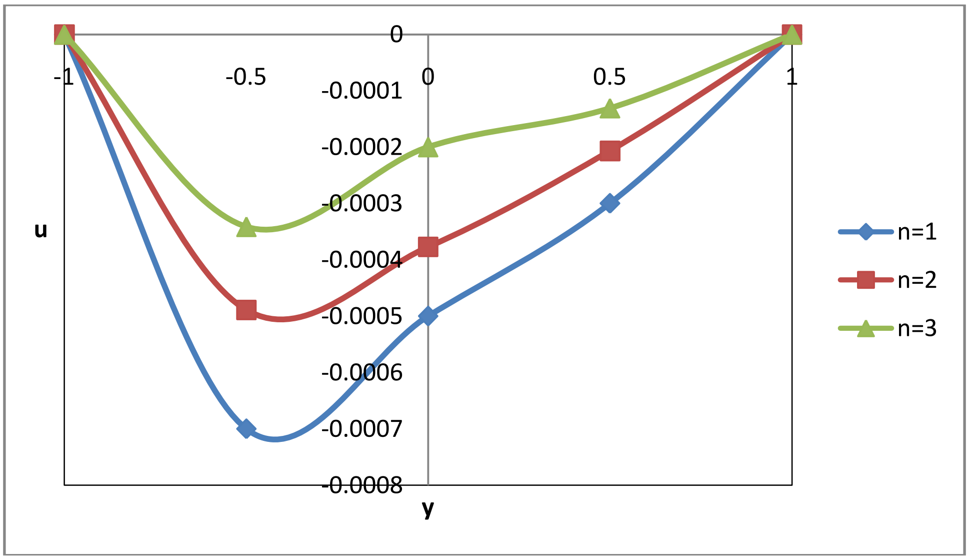

Fig. 2 The variation of velocity u with decay parameter n for Πρ = 0.25 , M = 2 ανδ τ=0.01.

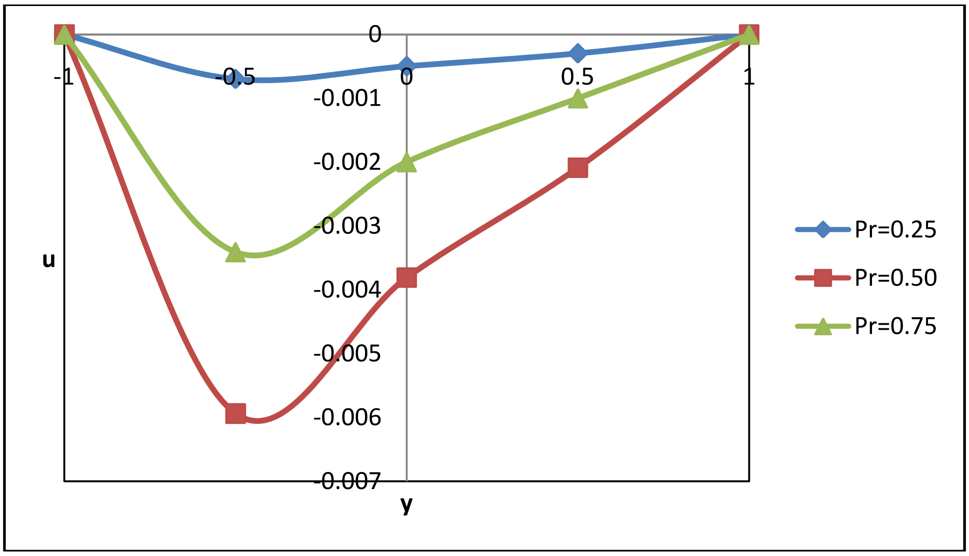

Fig. 3 The variation of velocity u with Prandtl number Pr for ν=1 , M = 2 ανδ τ=0.01.

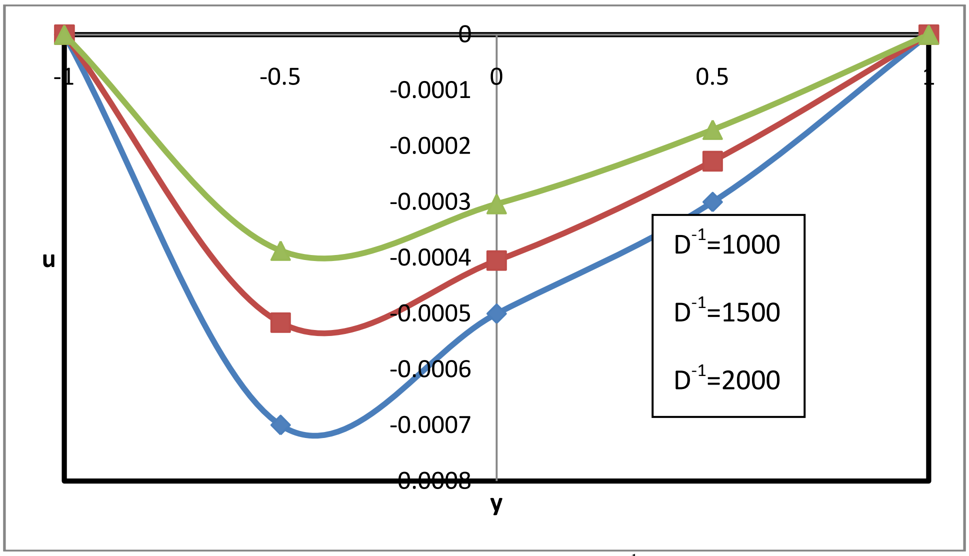

Fig. 4 The variation of velocity u with Darcy parameter Δ^{-1} φορ $\nu=1$, $M=2$ ανδ $\tau=0.01$.

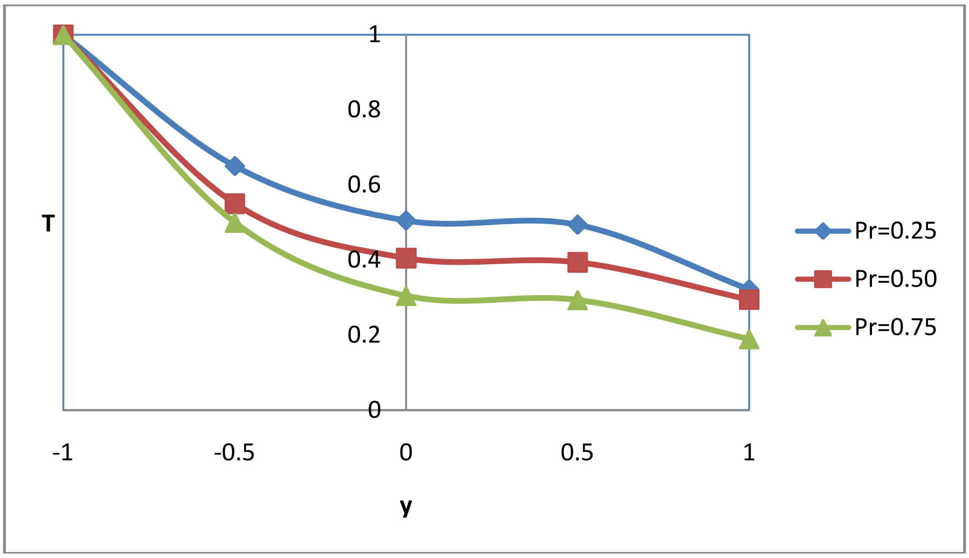

Fig. 5 The variation of temperature T with Prandtl number Pr φορ $\nu=1$ ανδ $\tau=0.01$.

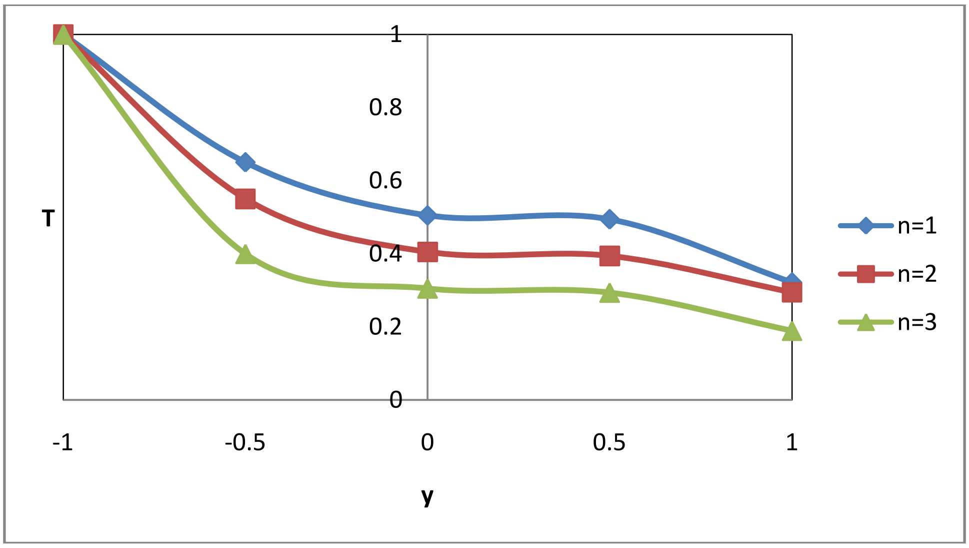

Fig. 6 The variation of temperature T with decay parameter n for Πρ = 0.25 ανδ $\tau=0.01$.

References

[1] Chakrabarty, S. and Borkakati, A. K. (2000): *Unsteady free convection MHD flow between two heated vertical parallel plates*, Pure and Applied Mathematika Science, 1(1-2).
[2] Davidson, P. A. (2001): *An Introduction to Magnetohydrodynamics*, Cambridge University Press.
[3] Gourla, M. G. and Katoch, S. (1991): *Unsteady free Convection MHD flow between heated vertical playes*, Ganita, 42(2), pp. 143-154.
[4] Sharma, P. R. and Kumar, N. (1998): *Unsteady Flow and Heat Transfer between two horizontal plates in the presence of transverse magnetic field*, Bulletin of Pure and Applied Sciences, 17E (1), pp. 39-49.
[5] Sreekant, S. *et al*. (2001): *Unsteady Free Convection Flow of an Incompressible Dissipative Viscous Fluid Past an an Infinite Vertical Plate*, Indian Journal of Pure and Applied Mathematics, 32 (7), pp.